

Convection and oscillations

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Received ??, accepted ??

Published online later

Key words convection – turbulence – stars: oscillations – Sun: oscillations – Sun: interior

In this short review on stellar convection dynamics I address the following, currently very topical, issues: (1) the surface effects of the Reynolds stresses and nonadiabaticity on solar-like pulsation frequencies, and (2) oscillation mode lifetimes of stochastically excited oscillations in red giants computed with different time-dependent convection formulations.

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1 Introduction

Seismic data of solar-type oscillations in distant stars are being provided with high precision by the space missions CoRoT (Baglin et al. 2009) and Kepler (Borucki et al. 2009; Christensen-Dalsgaard 2007), and hopefully also soon from the ground-based Danish SONG project (Grundahl et al. 2010). In contrast to solar observations the observed oscillation modes in distant stars are of only low spherical degree l . However, these are the modes which provide information from the deepest layers of the star. Some theoretically important properties, such as the gross structure of the energy-generating core and the extent to which it is convective, and the large-scale variation of the angular velocity, will become available. Such information will be of crucial importance for checking, and then calibrating, the theory of the structure and evolution of stars.

Solar-type oscillations are characterized by being intrinsically damped and stochastically driven by the near-surface turbulent velocity field (e.g., Goldreich & Keeley 1977, Balmforth 1992b, Houdek et al. 1999, Samadi et al. 2008). The turbulent convection produces acoustic noise in a broad frequency range, thereby exciting many of the global solar-type oscillations to observable amplitudes. The resulting rich acoustic power spectrum makes these modes particularly interesting for studying fluid-dynamical aspects of the stellar interior, and it is our hope that their mode properties can be used to probe the still ill-understood near-surface regions. The physics of these near-surface regions is, besides many other effects, further complicated by the dynamics of the transport of the turbulent fluxes of heat and momentum and their coupling to the pulsations. Several attempts have been made in the past to model the effects of convection dynamics on the oscillation frequencies, for which a time-dependent convection formalism is required. The influence of the momentum flux and its pulsationally

induced perturbation upon adiabatic and nonadiabatic pulsation frequencies were investigated first by Gough (1984), using a local time-dependent mixing-length formulation for convection (Gough 1977a), but a simplified analytical approximation to the eigenfunctions in the atmosphere. Gough concluded that the inclusion of the momentum flux in the mean model reduces the frequency residuals between observations and model computations, whereas the additional dynamical effects through their pulsationally induced perturbations actually aggravates this frequency discrepancies. Balmforth (1992a) studied these effects in a more consistent way and by means of the more sophisticated nonlocal time-dependent mixing-length formulation by Gough (1977b). Balmforth also concluded that the modification of the mean stratification by the momentum flux substantially decreases the adiabatic frequency residuals, whereas the effects of the momentum flux perturbation and nonadiabaticity lead to an increase in the modelled oscillation frequencies.

In Section 2 we discuss the results of more recent attempts to model the effects of the near-surface layers on the oscillations frequencies in the Sun and other stars. The other topical issue on the comparison between observed p-mode lifetimes in red giants and theoretical predictions, obtained with different time-dependent convection models, is addressed in Section 3.

2 The effect of convection dynamics on the oscillation frequencies

The near-surface layers in the Sun and other stars are still poorly understood, mainly because of the rather difficult to model complex physics of the interaction of convection, radiation, magnetism and rotation. Typical stellar structure calculations treat these superficial layers with simplified atmosphere models and the stratification of the superadiabatic region by means of a mixing-length approach (e.g. Böhm-Vitense 1958). Furthermore, the effects of the mean Reynolds stresses on the hydrostatic equilibrium are essen-

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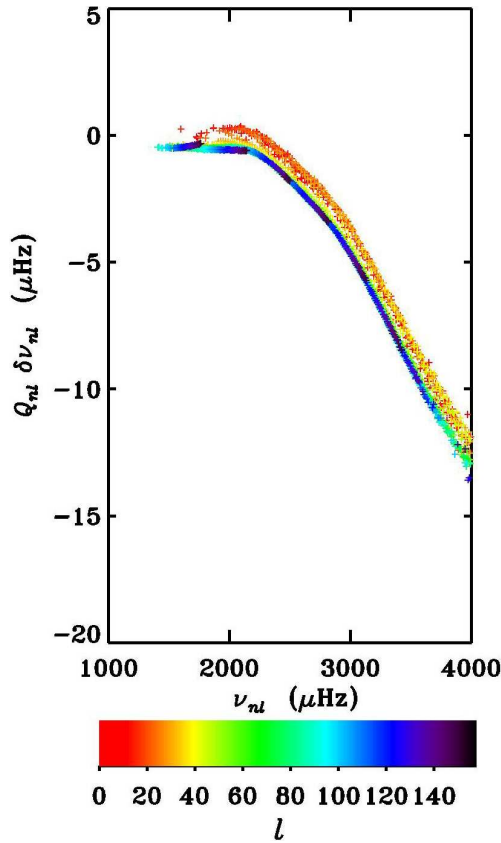


Fig. 1 Scaled differences between observed GONG frequencies and adiabatically computed frequencies of the ‘standard’ solar Model S. The degree l of the oscillation modes is indicated by the colour bar (from Christensen-Dalsgaard et al. 1996).

tially always ignored. Linear pulsation calculations typically adopt the adiabatic approximation and ignore the momentum flux perturbations (turbulent pressure fluctuations). However, nonadiabatic effects and the fluctuations of the turbulent fluxes (heat and momentum) do modify the modelled pulsation eigenfunctions and consequently also the oscillation frequencies (e.g., Gough 1980, Balmforth 1992b, Houdek 1996).

These effects are predominantly confined to the upper-most stellar layers, where the vertical scale of low- or intermediate degree modes is much less than the horizontal scale and when l is low the mode inertia is quite insensitive to degree l . Consequently the observed oscillation properties of low-degree modes depend little on the value of l (e.g., Libbrecht 1986). Moreover, the influence of the near-surface effects on the oscillation frequencies of low-degree modes is predominantly a function of frequency alone (e.g., Christensen-Dalsgaard & Thompson 1997), whereas for modes with intermediate degree a weak degree dependence is observed. This degree dependence, however, can be described by a simple scaling $Q_{nl} = I_{nl}/\bar{I}_0$ with mode inertia I_{nl} (e.g., Aerts, Christensen-Dalsgaard & Kurtz 2010, §7.1.4), where n is the radial mode order, and $\bar{I}_0(\nu)$ is obtained from inter-

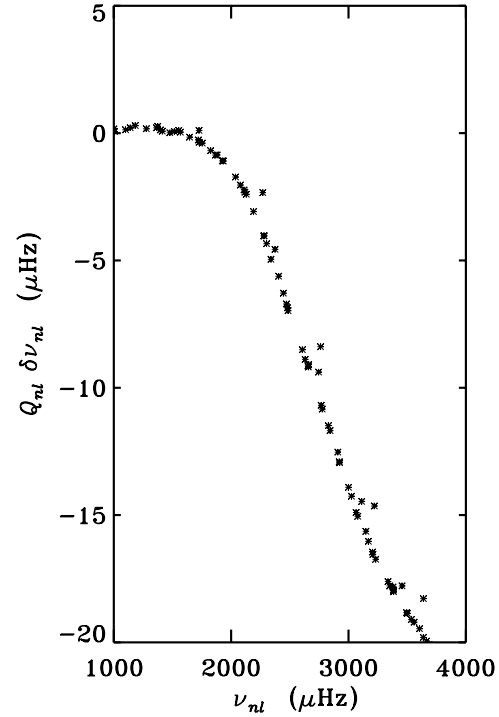


Fig. 2 Scaled adiabatic frequency differences between a model for which the near-surface layers were represented by a hydrodynamical simulation, and the ‘standard’ solar Model S (from Rosenthal et al. 1995).

polating the radial mode inertia I_{n0} to the nonradial oscillation frequency ν_{nl} . For a ‘standard’ solar model, such as Model S (Christensen-Dalsgaard et al. 1996), the (scaled) differences between the solar and model frequencies are indeed dominated by the near-surface effects and are predominantly a function of frequency alone. Differences between frequencies observed with the GONG instruments and adiabatically computed Model S frequencies are illustrated in Fig. 1. The increase of the frequency residuals with oscillation frequency depends on the modelling details of the functional form of the acoustic cutoff frequency ν_{ac} (as it affects the acoustic potential) with radius in the near-surface layers. For an isothermal atmosphere $\nu_{ac} = c/4\pi H$, where c is the adiabatic sound speed and H the pressure scale height. In the Sun $\nu_{ac} \simeq 5.5$ mHz. Thus, we see from Fig. 1 that the influence of the near-surface layers is most important, when the oscillation frequency ν is comparable to the acoustic cutoff frequency ν_{ac} . The cutoff frequency determines the location at which an incident acoustic wave is reflected back into the stellar interior, and the lower the frequency ν , the deeper the location at which this reflection takes place. For modes with frequencies ν much less than ν_{ac} reflection takes place so deep in the star that the modes are essentially unaffected by the near-surface structure. When ν is comparable with ν_{ac} , however, the inertia of the near-surface layers is a considerable fraction of the total mass above the reflecting layer, leading to a greater modification to the phase shift in the spatial oscillation eigenfunctions, and, through the

dispersion relation, also to a change in frequency. The inertia of the essentially hydrostatically moving near-surface layers depends on mass and consequently on the equilibrium pressure near the photosphere.

2.1 The effect of the Reynolds stresses in the mean structure

From the discussion before we conclude that the details of modelling the hydrostatic equilibrium structure in the near-surface layers play an important role in describing the residuals between observed and modelled oscillation frequencies, particularly for modes with ν close to ν_{ac} . Almost all stellar model calculations consider only the gradient of the gas pressure p_g in the equation of hydrostatic support. In the convectively unstable surface layers, however, the turbulent velocity field \mathbf{u} contributes to the hydrostatic support via the Reynolds stresses $\langle \rho \mathbf{u} \mathbf{u} \rangle$ (angular brackets denote an ensemble average), the (r, r) component $p_t := \langle \rho u_r u_r \rangle$ of which, acts as a pressure term additionally to the gas pressure p_g (e.g., Gough 1977ab). Solar models indicate that the turbulent pressure p_t can be as large as 15% of the total pressure $p = p_g + p_t$, as illustrated in Fig. 3 for three different simulations and models of the Sun.

Hydrodynamical simulations of stellar convection have enabled us to estimate the turbulent pressure (see Fig. 3). Rosenthal et al. (1995) investigated the effect on adiabatic eigenfrequencies of the contribution that the turbulent pressure makes to the mean hydrostatic stratification: he examined a hydrodynamical simulation by Stein & Nordlund (1991) of the outer 2% by radius of the Sun, matched continuously in sound speed to a model envelope calculated, as in a ‘standard’ solar model, with a local mixing-length formulation. The resulting frequency shifts of adiabatic oscillations between the simulations and the ‘standard’ solar reference model, Model S, are illustrated in Fig. 2. The frequency residuals behave similarly to the data in Fig. 1, though with larger shifts at higher oscillation frequencies.

If turbulent pressure is considered in the mean structure, additional assumption have to be made about the turbulent pressure perturbation in the adiabatic pulsation equations. The first adiabatic exponent $\gamma_1 = (\partial \ln p_g / \partial \ln \rho)_s$ (s being the specific entropy), is a purely thermodynamic quantity and is expressed by means of the gas pressure p_g . Consequently, in the presence of turbulent pressure p_t , such that the total pressure $p = p_g + p_t$ satisfies the equation of hydrostatic equilibrium, γ_1 experiences a modification of the form

$$\tilde{\gamma}_1 = \left(\frac{\partial \ln p}{\partial \ln \rho} \right)_s = \frac{1}{p} \left[\left(\frac{\partial p_g}{\partial \ln \rho} \right)_s + \left(\frac{\partial p_t}{\partial \ln \rho} \right)_s \right] \quad (1)$$

and the Lagrangian perturbation in the total pressure is

$$\frac{\delta p}{p} = \frac{\delta p_g}{p_g} + \frac{\delta p_t}{p} = \tilde{\gamma}_1 \frac{\delta \rho}{\rho}. \quad (2)$$

Nonadiabatic pulsation calculations with the inclusion of the turbulent pressure fluctuation δp_t (Houdek 1996), and

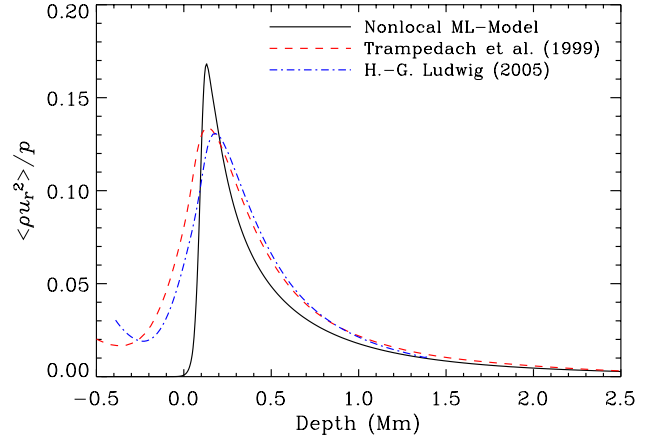


Fig. 3 Turbulent pressure $p_t = \langle \rho u_r^2 \rangle$ over total pressure $p = p_g + p_t$ as a function of the depth variable $R_\odot - r$ for various solar simulations and models (R_\odot is the photospheric solar radius). Results are shown for the nonlocal mixing-length model by Gough (1977ab, solid curve) and from hydrodynamical simulations by Trampedach (1999, personal communication, dashed curve) and Ludwig (2005, personal communication, dot-dashed curve).

hydrodynamical simulation results (Rosenthal et al. 1995) indicate that δp_t varies approximately in quadrature with the other terms in the linearized momentum equation, and hence contributes predominantly to the imaginary part of the frequency shift, i.e. to the linear damping rate. The Lagrangian perturbation δp_g , however, responds adiabatically. Therefore δp_t can be neglected in equation (2), i.e. in the calculation of the real adiabatic eigenfrequencies it is assumed that $\delta p/p \simeq \delta p_g/p \simeq \tilde{\gamma}_1 \delta \rho / \rho$. With this assumption $\tilde{\gamma}_1 \simeq (p_g/p) \gamma_1$, and the only modification to the adiabatic oscillation equations is the replacement of γ_1 by $\tilde{\gamma}_1$ (Rosenthal et al. 1995).

2.2 The effects of nonadiabaticity and momentum flux perturbation

The effects of nonadiabaticity and convection dynamics on the pulsation frequencies were, for example, studied by Balmforth (1992b), Rosenthal et al. (1995) and Houdek (1996). In these studies the nonlocal, time-dependent generalization of the mixing-length formulation by Gough (1977ab) was adopted to model the heat and momentum flux consistently in both the equilibrium envelope model and in the nonadiabatic stability analysis. Houdek (1996) considered the following models:

- L.a A local mixing-length formulation without turbulent pressure p_t was used to construct the mean envelope model. Frequencies were computed in the adiabatic approximation assuming $\delta p_t = 0$ (see Section 2.1).
- NL.a Gough's (1977ab) nonlocal, mixing-length model, including turbulent pressure, was used to construct the mean envelope model. Frequencies

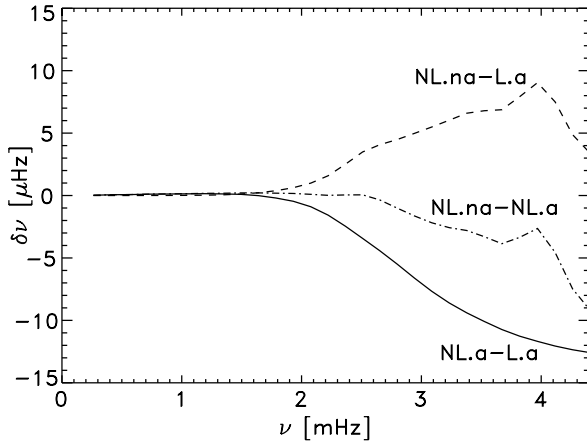


Fig. 4 Frequency residuals between solar models calculated with a nonlocal generalization and the 'standard' local mixing-length formulation. The solid curve (NL.a-L.a) shows the adiabatic radial oscillation frequency shifts, under the assumption $\delta p_t = 0$, caused by the turbulent pressure p_t in the mean envelope. The dashed curve (NL.na-NL.a) is the frequency shift caused by nonadiabaticity and effects of including consistently δp_t . The overall frequency shift (NL.na-L.a) is plotted by the dot-dashed curve (from Houdek 1996).

were computed in the adiabatic approximation assuming $\delta p_t = 0$.

NL.na The mean envelope model was constructed as in NL.a. Nonadiabatic frequencies were computed including consistently the Lagrangian perturbations of the convective heat flux δF_c and turbulent momentum flux δp_t .

Additional care was necessary when frequencies between models with different convection treatments were compared, such as in the models L.a and NL.a. In order to isolate the effect of the near-surface structures on the oscillation frequencies the models had to possess the same stratification in their deep interiors. This was obtained by requiring the models to lie on the same adiabat near the base of the (surface) convection zone and to have the same convection-zone depth. Varying the mixing-length parameter $\alpha = H/\ell$ (ℓ is the mixing length) and hydrogen abundance by iteration in model L.a, the same values for temperature and pressure were found at the base of the convection zone than those in models NL.a and NL.na. The radiative interior of the nonlocal models NL.a and NL.na were then replaced by the solution of the local model L.a, and the convection-zone depth was calibrated to $0.287 R_\odot$ (Christensen-Dalsgaard, Gough & Thompson 1991). Further details of the adopted physics in the model calculations can be found in Houdek et al. (1999).

The outcome of these calculations is shown in Fig. 4. As for the hydrodynamical simulations (Fig. 2) the effect of the Reynolds stresses in the mean structure decreases the adiabatic frequencies (NL.a-L.a, solid curve) for frequencies larger than about 2 mHz, though the maximum deficit

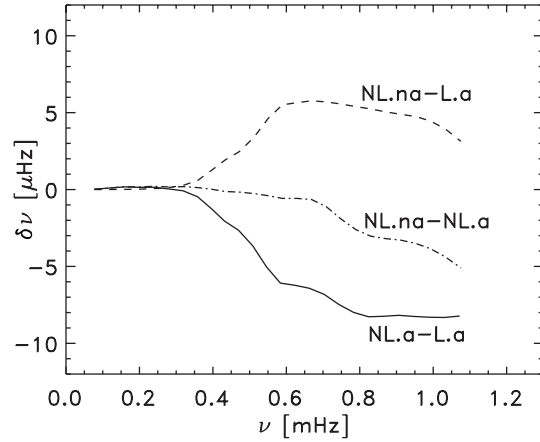


Fig. 5 Frequency residuals between models for the solar-like star η Boo. The model calculations and line styles are as in Fig. 4 (from Houdek 1996).

of about $12 \mu\text{Hz}$ is smaller than in the hydrodynamical simulations. The effects of nonadiabaticity and δp_t (NL.na-L.a, dashed curve), however, lead to an increase of the mode frequencies by as much as $\sim 9 \mu\text{Hz}$, nearly cancelling the downshifts from the effect of p_t in the mean structure, as illustrated by the dot-dashed curve (NL.na-NL.a). If the positive frequency shifts between models NL.na and L.a (dotted curve) are interpreted as the nonadiabatic and momentum flux corrections to the oscillation frequencies then their effects are to bring the frequency residuals of the hydrodynamical simulations (Fig. 2) in better agreement with the data plotted in Fig. 1. The effects of the near-surface regions in the Sun were also considered by Rosenthal et al. (1999) and Li et al. (2002) based on hydrodynamical simulations.

A similar conclusion as in the solar case was also found for the solar-like star η Boo by Christensen-Dalsgaard et al. (1995), Houdek (1996), demonstrated in Fig. 5, and more recently by Straka et al. (2006).

The near-surface frequency corrections also affect the determination of the modelled mean large frequency separation $\Delta\nu := \langle \nu_{n+1l} - \nu_{nl} \rangle$ (angular brackets indicate an average over n and l). In both models for the Sun and for η Boo the resulting corrections to $\Delta\nu$ are about $-1 \mu\text{Hz}$. Although this correction is less than 1% it does affect the determination of the stellar radii and ages from the observed values of $\Delta\nu$ and small frequency separation $\delta\nu_{02}$ in distant stars. A simple procedure for estimating the near-surface frequency corrections was suggested recently by Kjeldsen et al. (2008), based on the *ansatz* that the frequency shifts can be scaled as $a(\nu/\nu_0)^b$ (Christensen-Dalsgaard & Gough 1980), where ν_0 is a suitable reference frequency, a is determined from fitting this expression to the observed frequencies, and b is obtained from the solar data (see also Christensen-Dalsgaard, these proceedings). It is to be hoped that the analyses of high-quality data for a broad range of stars, observed from the space mission Kepler, will contribute to a better understanding of the near-surface effects.

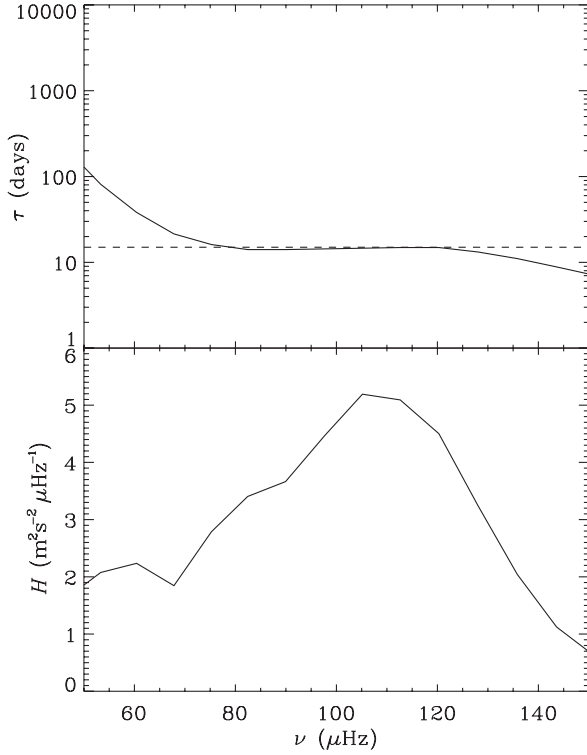


Fig. 6 Radial pulsation mode properties of solar-type oscillations for a model of the red-giant star ξ Hydrae. The computations adopted Gough's (1977) time-dependent convection model. Top: theoretical mode lifetimes as a function of pulsation frequency. The horizontally dashed line indicates a mode lifetime of 15 days. Bottom: modelled oscillation mode heights as a function of pulsation frequency (from Houdek & Gough 2002).

3 Mode lifetimes in red giants

The first convincing detection of solar-type oscillations in a red-giant star was announced by an international team of astronomers (Frandsen et al. 2002) for the star ξ Hydrae (G7III) with velocity oscillation amplitudes of about 2 ms^{-1} . Houdek & Gough (2002) calculated mode properties for the red-giant star ξ Hydrae and reported velocity amplitudes that were in good agreement with the observations. Moreover, using Gough's (1977ab) time-dependent convection model, these authors estimated theoretical mode lifetimes and reported for the most prominent p modes a lifetime τ of about 15–17 days (see Fig. 6). This prediction was, however, later challenged by Stello et al. (2004, 2006), who developed a new method for measuring mode lifetimes from various properties of the observed oscillation power spectrum and reported a measured mode lifetime of only 2–3 days for the star ξ Hydrae. This is in stark contrast to the predicted values by Houdek & Gough (2002), a discrepancy that needs to be understood. In the new method by Stello et al. it is assumed that the observed oscillation power spectrum is dominated by radial modes, because nonradial modes have larger inertiae I and

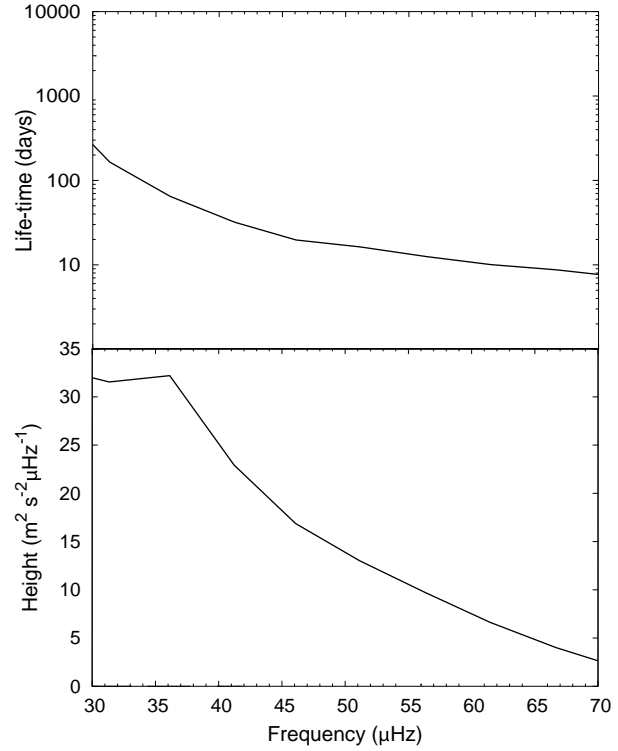


Fig. 7 Radial pulsation mode properties of solar-type oscillations for a red-giant model in the core helium-burning phase. The computations were carried out by Dupret et al. (2009) using the convection model by Grigahcène et al. (2004). Top: theoretical mode lifetimes as a function of pulsation frequency. The radial mode lifetimes decrease monotonically with frequency. Bottom: modelled oscillation mode heights as a function of pulsation frequency (adopted from Dupret et al. 2009).

consequently lower amplitudes V (e.g. Dziembowski et al., 2001, Christensen-Dalsgaard 2004). The maximum peaks in the observed power spectrum, however, which are actually mode heights H and related to the velocity amplitudes V via the relation $V^2 = \eta H/2$ (e.g., Chaplin et al. 2005), in which $\eta = \tau^{-1}$ is the damping rate in units of angular frequency, are independent of the pulsation mode inertiae. Consequently the pulsation heights H of nonradial modes could have similar values to those of radial modes of similar frequencies, provided the modes are resolved (e.g. Dupret et al. 2009). A first hint, shedding some light on the pulsation mode lifetimes in red giants, has been provided by oscillation data obtained by CoRoT (e.g., De Ridder et al. 2009) from several red-clump stars, and by Kepler (Bedding et al. 2010). These data support the possibility of the presence of nonradial pulsation modes (see also Hekker, these proceedings), bringing observed mode lifetimes and theoretical predictions in better agreement (see also Carrier et al. 2010).

Fig. 6 illustrates the predictions of radial mode lifetimes and oscillation heights for a model of the red giant star ξ Hydrae. As in the Sun the predicted mode lifetimes in ξ Hydrae show a pronounced plateau about the maximum pulsation height

near $105 \mu\text{Hz}$, a frequency value that is in good agreement with the observations by Frandsen et al. (2002).

Red-giant stars may display different frequency patterns in the oscillation power spectrum, from regularly to very complicated patterns, depending on the density contrast between the core and the envelope (e.g., Dupret et al. 2009). Evolutionary calculations suggest that ξ Hydrae is most likely in the core helium-burning phase (Teixeira et al. 2003; Christensen-Dalsgaard 2004). Dupret et al. (2009) computed mode lifetimes and pulsation heights for several red-giant models in different evolutionary phases. Mode properties of radial pulsations for a red-giant model in the core helium-burning phase, calculated by Dupret et al., are displayed in Fig. 7. Although the results in Figs 6 and 7 are for different models, though both models are in the core helium-burning phase, it is here assumed that their pulsation properties can be compared qualitatively. The most interesting qualitative difference between Figs 6 and 7 is the frequency dependence of the radial mode lifetime (top panels) showing a monotonic decrease with frequency in the results by Dupret et al. This could be a result of having adopted different convection formulations in the model computations. Also, the frequency dependence of the pulsation mode heights (bottom panels) are different. In the stability computations by Dupret et al. (Fig. 7) the turbulent fluxes were modelled according to the time-dependent convection formulation by Grigahcène et al. (2004), which is based on Unno's (1967) formulation. Houdek & Gough (2002) adopted Gough's (1977ab) time-dependent nonlocal convection formulation in the model computations for ξ Hydrae (Fig. 6). Stability computations of red-giant oscillations were also recently addressed by Xiong & Deng (2007) using Xiong's (1989) time-dependent convection formulation. Some of the model differences between Unno's (1967) and Gough's (1977a) convection formulation were discussed by Houdek (2008). It is to be hoped that the high-quality data from, for example Kepler, will not only help modellers to calibrate but also to improve convection models for pulsating stars.

Acknowledgements. I thank the referee for his helpful comments. Support by the Austrian Science Fund (FWF project P221205) is thankfully acknowledged.

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